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Finite-Time Model Predictive Control for Nonlinear Discrete-Time Semi-Markov Jump Systems

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Introduction

This paper proposes a finite-time model predictive control (MPC) strategy for discrete-time semi-Markov jump nonlinear systems (s-MJSs) to address complex sojourn-time distributions and nonlinear dynamics. By introducing a semi-Markov kernel (SMK) to characterize mode transitions and employing incremental quadratic constraints (IQC) to bound nonlinearities, the method ensures stochastic finite-time stability (SFTS) under disturbances. The MPC framework minimizes control inputs while satisfying stability constraints, the effectiveness of which has been validated through numerical simulations. Results demonstrate the effectiveness of the approach in achieving stochastic finite-time stability with improved transient performance.

Research Questions

1) Developing a stability-guaranteed control framework for nonlinear semi-Markov jump systems with arbitrary sojourn-time distributions, overcoming geometric/exponential constraints of conventional MJSs.

2) Constructing a model predictive control framework that jointly addresses nonlinear model, stochastic switching, and input constraints through synergistic integration of incremental quadratic constraints and semi-Markov kernel.

3) Demonstrating enhanced transient performance of the proposed finite-time MPC in short-duration operations (e.g., power system switching, robotic fail-safe control) compared to asymptotic stabilization approaches.

Methodologies

1) Develop MPC framework using semi-Markov kernel to handle arbitrary sojourn-time distributions, ensuring stochastic finite-time stability for nonlinear s-MJSs.

2) Integrate IQC with SMK in MPC to jointly address nonlinear dynamics, stochastic switching, and input constraints via SDP optimization.

3) Guarantee transient performance with finite-time stability (validated numerically), outperforming asymptotic methods in short-duration operations.

Mathematical Formulas

Discrete-time s-MJSs:

$$x_{k+1} = A_{r_k} x_k + B_{r_k} u_k + q_k + \omega_k .$$

semi-Markov kernel

transition probabilities (TPs)

$$\theta_{pq} \triangleq \Pr\left(M_{n+1} = q \middle| M_n = p\right)$$

probability density function (PDF)

$$\psi_{pq}(\tau) \triangleq \Pr\left(S_{n+1} = \tau \middle| M_{n+1} = q, M_n = p\right)$$

discrete-time SMK

$$\pi_{pq}(\tau) = \frac{\Pr(M_{n+1} = q, M_n = p)}{\Pr(M_n = p)} \frac{\Pr(M_{n+1} = q, S_{n+1} = \tau, M_n = p)}{\Pr(M_{n+1} = q, M_n = p)} = \theta_{pq}\psi_{pq}(\tau).$$

incremental quadratic constraint

$$\begin{bmatrix} x_k \\ q_k \end{bmatrix}^T \mathbb{Q} \begin{bmatrix} x_k \\ q_k \end{bmatrix} \geq 0 .$$

$$x_k^T Q_{11} x_k + x_k^T Q_{12} q_k + q_k^T Q_{21} x_k + q_k^T Q_{11} q_k \geq 0 .$$

stochastic finite-time stability

$$E\left\{x_0^T G_{r_{k_0}} x_0\right\} \leq c_1 \Rightarrow E\left\{x_k^T G_{r_k} x_k\right\}, \forall r_k \in \Phi, \forall k \in \{1, 2, ..., H\}$$

Model Predictive Control

cost function:

$$J_k = \sum_{f=k}^{H-1} u_{f|k}^T Y_{r_{f|k}} u_{f|k} .$$

optimization condition:

$$\min_{F_p, P_p} \gamma$$

$$s.t. J_k < \gamma,$$

$$V\left(x_{k+1}, r_{k_n}\right) \leq V\left(x_k, r_{k_n}\right),$$

$$E\left\{V\left(x_{k_{n+1}}, r_{k_{n+1}}\right)\right\} \leq E\left\{V\left(x_{k_n}, r_{k_n}\right)\right\},$$

$$\frac{\lambda_{\max}(P_p)}{\lambda_{\min}(P_p)} x_k^T G_p x_k \leq c_2 .$$

Figures

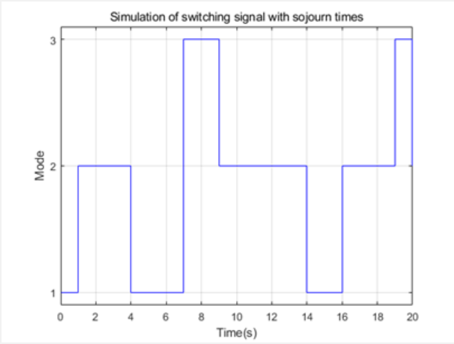

Figure 1. Simulation of switching signal

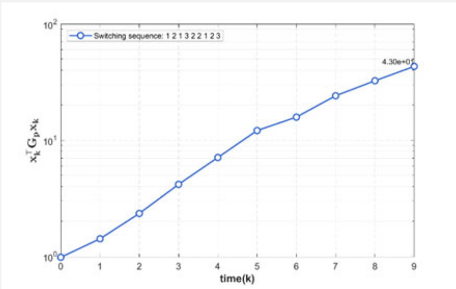

Figure 2. Open-loop s-MJSs

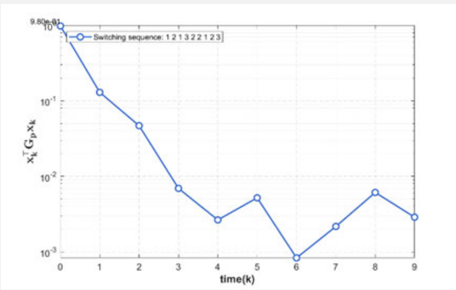

Figure 3. Closed-loop s-MJSs

Conclusion

This paper develops a finite-time model predictive control (MPC) strategy for discrete-time semi-Markov jump nonlinear systems (s-MJSs). By introducing the multi-step semi-Markov kernel and incremental quadratic constraints, the proposed method effectively handles the complex sojourn-time distributions and system nonlinearities. Theoretical analysis demonstrates that the designed controller ensures stochastic finite-time stability of the closed-loop system, even in the presence of disturbances. Simulation results validate the effectiveness of the proposed approach, highlighting its potential for practical engineering applications involving complex switching systems.

Mathematical Formulas

core formula of LMI

$$\min_{\lambda_1,\lambda_2,Z_p,U_p,\mathrm{H}_p,\tilde{\mathrm{H}}_p(t)} \gamma$$

$$s.t.\begin{bmatrix} Z_p+Z_p^T-\mathrm{H}_p & x_k \\ * & \gamma \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} -H_p & 0 & 0 & \left(A_pZ_p+B_pU_p\right)^T & U_p^T & Z_p^T & Z_p^T & 0 \\ * & Q_{22} & 0 & Z_p^T & 0 & 0 & 0 & I \\ * & * & 0 & Z_p^T & 0 & 0 & 0 & 0 \\ * & * & * & -Z_p^T-Z_p+H_p & 0 & 0 & 0 & 0 \\ * & * & * & * & -Y_p^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & -Q_{11}^{-1} & 0 & 0 \\ 0 & I & * & * & * & * & -Q_{12}^{-1} & 0 \\ Z_p & 0 & * & * & * & * & * & -Q_{21}^{-1} \end{bmatrix} \leq 0,$$

$$\mathrm{H}_p-\tilde{\mathrm{H}}_p(0)\geq 0,$$

$$\begin{bmatrix} \lambda_1^{-1}G_p^{-1} & Z_p \\ * & \mathrm{H}_p \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \lambda_2^{-1}G_p^{-1} & Z_p \\ * & Z_p+Z_p^T-\mathrm{H}_p \end{bmatrix} \leq 0,$$

$$\lambda_2^{-1}c_2-\lambda_1^{-1}x_k^TG_px_k\geq 0.$$

numerical example

$$A_1=\begin{bmatrix} 0.9347 & 0.5194 \\ 0.3835 & 0.8310 \end{bmatrix} \quad B_1=\begin{bmatrix} -1.4462 \\ -0.7012 \end{bmatrix}$$

$$A_2=\begin{bmatrix} 0.8365 & 0.4192 \\ 0.3836 & 0.6310 \end{bmatrix} \quad B_2=\begin{bmatrix} -1.3893 \\ -0.6512 \end{bmatrix}$$

$$A_3=\begin{bmatrix} 0.7758 & 0.6394 \\ 0.3875 & 0.6513 \end{bmatrix} \quad B_3=\begin{bmatrix} -1.7654 \\ -0.8213 \end{bmatrix}$$

transition probabilities

$$\Theta=\begin{bmatrix} 0 & 0.25 & 0.75 \\ 0.5 & 0 & 0.5 \\ 0.3 & 0.7 & 0 \end{bmatrix}$$

sojourn-time distributions

$$\Psi(\tau)=\begin{bmatrix} 0 & \dfrac{0.4^{\tau}0.6^{10-\tau}10!}{(10-\tau)!\tau!} & \dfrac{0.6^{\tau}0.4^{10-\tau}10!}{(10-\tau)!\tau!} \\ 0.7^{(\tau-1)^{1.5}}-0.7^{\tau^{1.5}} & 0 & 0.3^{(\tau-1)^{1.5}}-0.3^{\tau^{1.5}} \\ 0.5^{(\tau-1)^{1.3}}-0.5^{\tau^{1.3}} & 0.5^{(\tau-1)^{1.5}}-0.5^{\tau^{1.5}} & 0 \end{bmatrix}$$