

## Heading Optimization for Multi-AUV Target Tracking under Localization Uncertainty

Wen Xing<sup>1,2</sup>, Yijing Zhang<sup>1,\*</sup> and Yongzhen Sun<sup>2</sup>

<sup>1</sup>College of Intelligent Systems Science and Engineering, Harbin Engineering University, Harbin 150001, P. R. China

<sup>2</sup>Qingdao Innovation and Development Center, Harbin Engineering University, Qingdao 266000, P. R. China  
{ xingwen; syz0314 }@hrbeu.edu.cn; \*Corresponding author: hgcqjdj@hrbeu.edu.cn

### Introduction

In recent years, autonomous underwater vehicles (AUVs) have been widely deployed for underwater missions including target tracking and classification. Owing to the limited detection range and operational constraints of individual AUVs, collaborative multi-AUV systems have emerged as a prevalent research focus for target tracking applications. In underwater environments where global positioning system signals are unavailable and anchor-based localization incurs substantial deployment costs, AUVs in anchor-free configurations suffer from cumulative positioning errors over time. This growing positional uncertainty consequently increases estimation errors in target tracking. Furthermore, under bearing-only measurement conditions, the relative geometry between the target and AUV formation critically determines system observability, where enhanced observability improves both estimation accuracy and algorithm convergence rate. To address these challenges by improving tracking precision and mitigating error accumulation during prolonged AUV operations, it is essential to incorporate AUV position uncertainty into the tracking process and implement real-time heading optimization based on target state estimation.

### Research Questions

Extended underwater operations cause progressive degradation of AUV positioning accuracy due to error accumulation in inertial navigation systems (INS). Although multi-AUV cooperative positioning strategies can mitigate these errors, their effectiveness is fundamentally constrained by the inherent inaccuracy of individual AUV position measurement. This limitation occurs because fusing inaccurate positioning data propagates and compounds measurement errors, resulting in irreducible residual errors within the cooperative localization framework.

Target observability is fundamentally determined by the sensor-target geometry. Optimal sensor geometries – characterized by well-spaced AUVs surrounding the target at appropriate baselines – maximize the Fisher Information Matrix (FIM) determinant or trace, thereby enhancing estimation accuracy. Conversely, degenerate geometries (e.g., colinear AUV-target configurations) lead to rank-deficient observation matrices and consequently unobservable target states.

### Research Questions

To overcome these challenges, this study introduces

- 1) An equivalent measurement noise compensation method that accounts for AUV positioning uncertainty;
- 2) A heading optimization strategy that maximizes FIM-based observability metrics.

### Methodologies

This paper proposes a unified framework for simultaneous target tracking and heading optimization in multi-AUV systems. To address accumulated positioning errors during prolonged underwater navigation, we develop an equivalent measurement noise method that incorporates AUV position uncertainty into cooperative localization. Building on this foundation, we implement an interactive multiple-model unscented Kalman filter (IMM-UKF) to improve tracking robustness for dynamically maneuvering targets. Recognizing that sensor-target geometry fundamentally determines the FIM eigenvalue structure, we formulate an FIM-based cost function to quantify target observability. This cost function serves as the optimization criterion for AUV heading determination, with optimal headings computed using a branch-and-bound algorithm.

### Mathematical Formulas

- 1) Equivalent measurement noise method

Given the estimated AUV position, the measurement equation takes the form

$$Z_k^{i,j} = h_{i,j}(\hat{X}_k^i, X_k^j) + V_Z^i + V_k = h_{i,j}(\hat{X}_k^i, X_k^j) + V_k' \quad (1)$$

The measurement noise matrix is expressed as

$$\hat{R}_k^i = H_k^{i,j} P_{k|k-1}^i H_k^{i,jT} + R_k^i \quad (2)$$

- 2) IMM-UKF algorithm

The target state and covariance estimates are given by

$$\hat{X}_{k|k} = \sum_{p=1}^3 \hat{X}_{k|k}^p \mu_k^p \quad (3)$$

$$P_{k|k} = \sum_{p=1}^3 \mu_k^p \left[ P_{k|k}^p + (\hat{X}_{k|k}^p - \hat{X}_{k|k})(\hat{X}_{k|k}^p - \hat{X}_{k|k})^T \right] \quad (4)$$

## Mathematical Formulas

### 3) Heading optimization algorithm

The FIM-based cost function is defined as

$$S_k^* = \arg \min \text{trace} \left\{ J_{X_k^t} + \sum_{i=1}^{n_{\text{AUV}}} J_{Z_k^i} \right\} \quad (5)$$

## Figures

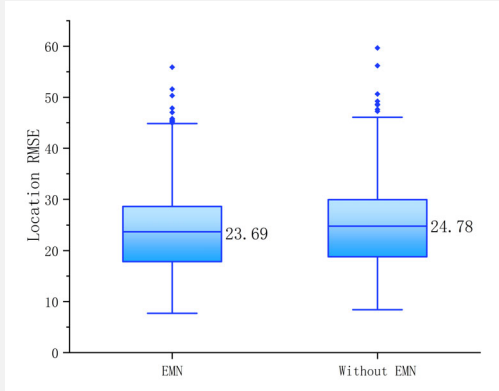


Figure 1. Comparison of tracking and positioning errors with/without equivalent measurement noise compensation

Figure 1 displays the positioning error distribution from 1000 Monte Carlo runs, comparing the proposed equivalent measurement noise method with conventional uncompensated approaches. Boxplot analysis demonstrates statistically significant improvements across all error metrics, with a 4.4% reduction in mean positioning error. These results quantitatively validate the effectiveness of the proposed method in mitigating underwater navigation errors.

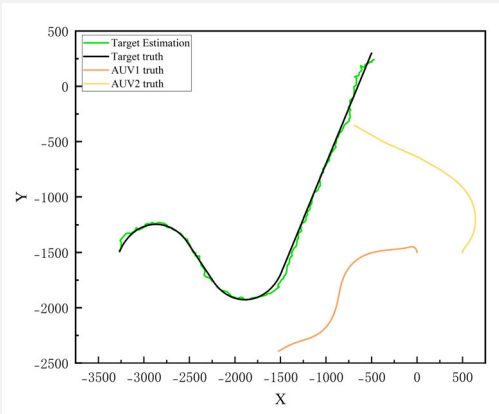


Figure 2. Optimal AUV trajectories and target tracking based on FIM maximization

## Figures

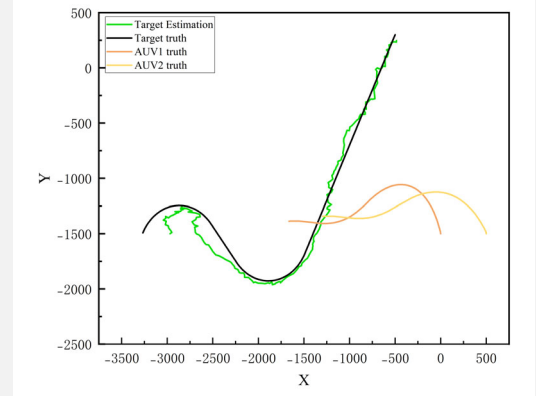


Figure 3. Optimal AUV trajectories and target tracking based on Euclidean distance minimization

Figures 2 and 3 respectively present the target state estimation results and optimized AUV trajectories under different optimization criteria. Figure 2 demonstrates the tracking performance and trajectory optimization achieved by the proposed FIM-based cost function, while Figure 3 shows the comparative results using the Euclidean distance (ED) metric, defined as

$$S_k^* = \arg \min \sum_{i=1}^{n_{\text{AUV}}} \left[ \left( \hat{x}_{k|k-1}^i - \hat{x}_{k|k-1}^i \right)^2 + \left( \hat{y}_{k|k-1}^i - \hat{y}_{k|k-1}^i \right)^2 \right]$$

Figures 2 and 3 compare the estimated target trajectories (green lines) with truth trajectories (black lines), demonstrating significantly lower tracking errors when using the FIM-based cost function compared to the ED approach. These results quantitatively validate that FIM-based optimization substantially enhances tracking accuracy in multi-AUV systems, reducing the mean position error by 82.13%.

## Conclusion

This paper presents a novel multi-AUV cooperative framework for simultaneous target tracking and formation optimization. The proposed system achieves robust tracking performance while actively improving estimation accuracy through dynamic heading adjustments. Our work makes three contributions: 1) An equivalent measurement noise compensation method that accounts for AUV positional uncertainty in cooperative localization, effectively mitigating navigation drift during extended underwater operations; 2) An IMM-UKF algorithm designed to handle complex target maneuver patterns through adaptive model switching; 3) An FIM-based optimization strategy that maximizes observability by optimizing AUV formation geometry. Extensive simulation studies demonstrate the framework's ability to maintain continuous target tracking with a 82.13% improvement in tracking accuracy compared to conventional approaches.